Robotics Digest

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1- Introduction to robotics

- Definitions
- Anthropomorphy

- Serial versus Parallel

Parallel Kinematics are kinematically stiffer (eigen frequency is higher), lighter, and ability to be faster
Serial Kinematics have more workspace

- Angular versus Linear

Angular variants are faster
Linear variants are stiffer, and more precise

- DOFs, Mobility and Redundancy

1- Introduction to robotics

- DOFs, Mobility and Redundancy
  - **DOF**: number of independent movements of the eof,
  - **Mobility**: is the number of actuators for serial K.
    is calculated with the formulas of Grübler or Kinematic loops for parallel K.
  - **Redundancy:** if the number of actuators > the DOF (valid for parallel K and Serial K)

- **Mobility** for parallel structures indicates if there is a potential for constrained movements (Overcontraints/ hyper-guidage) of internal mobilities.

- **6 DoFs parallel structures- Comparaison. Gough Stewart, Hunt and Artigue**

Degrees of freedom

Degrees of freedom (DOF) is the number of the (geometrically) independent variables required to define the end effector pose of a robot

Maximum allowed DOFs are 6 $\{x, y, z, \theta x, \theta y, \theta z\}$ *at the end effector (tool of robot)*

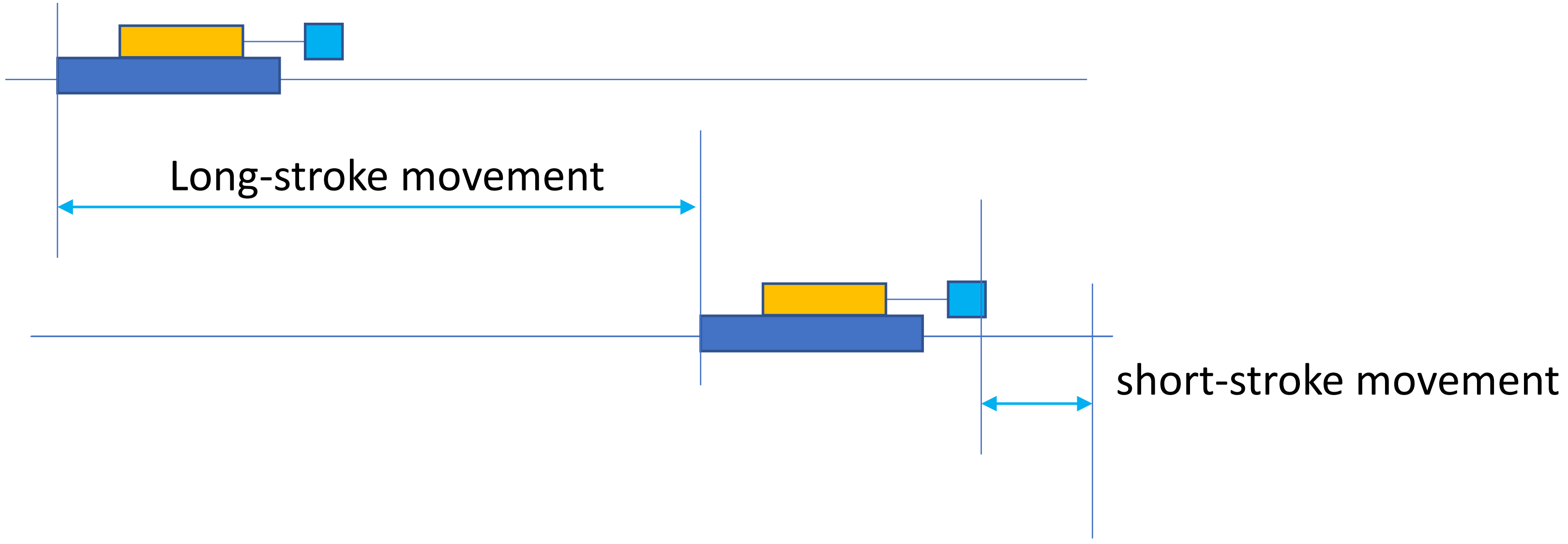
x *if the translation x is possible*
y *if the translation y is possible*

And so on with

Translation ***z***
Orientation $\boldsymbol{\theta x}$,
Orientation $\boldsymbol{\theta y}$,
Orientation $\boldsymbol{\theta z}$

The robot is redundant if the number of actuators > DOF

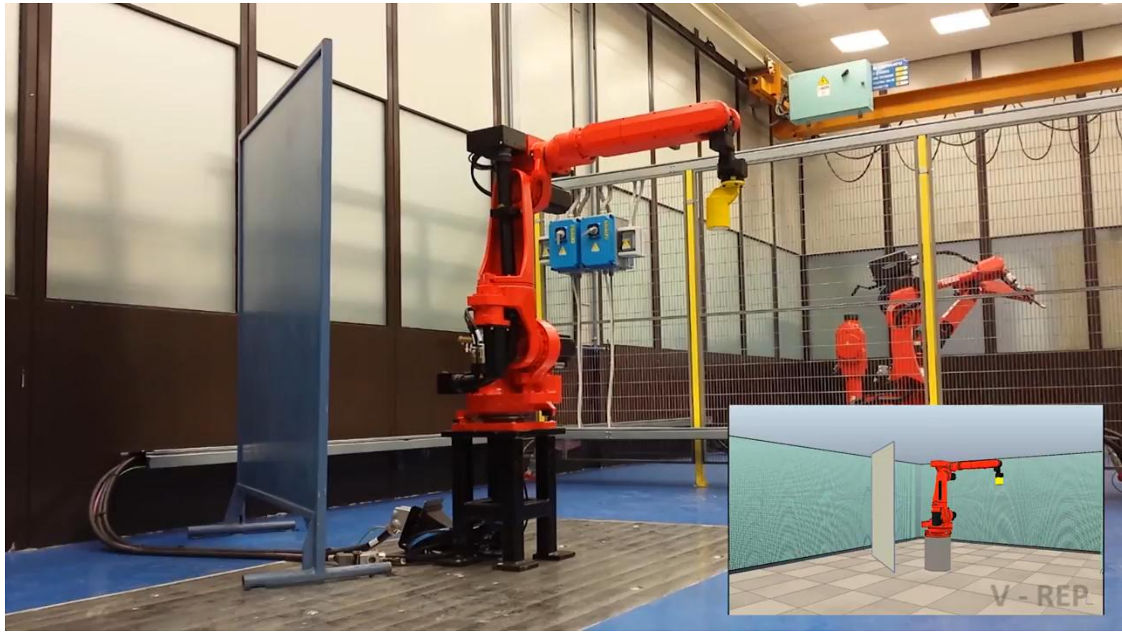
Example



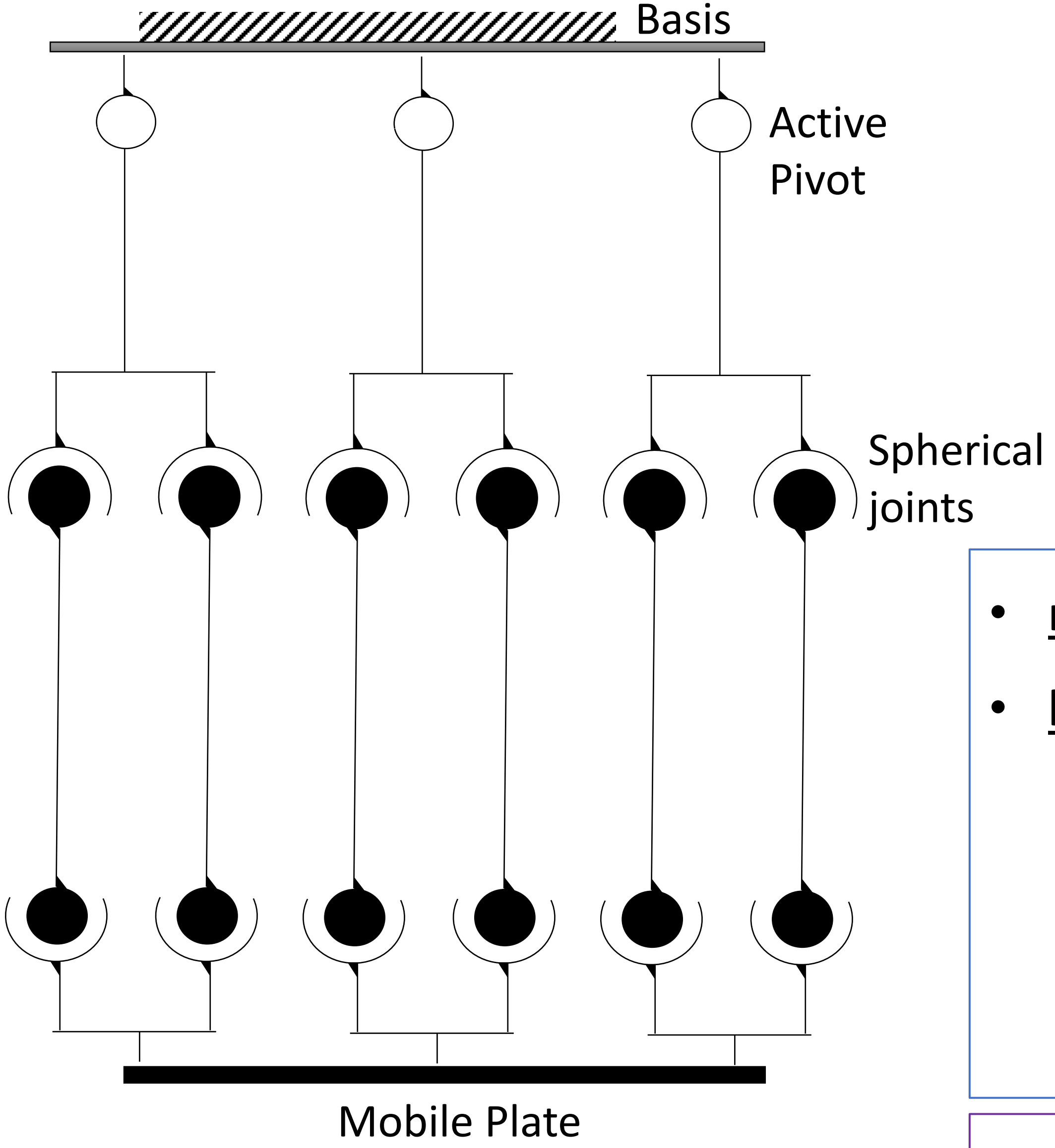
The robot is redundant if the number of actuators > DOF

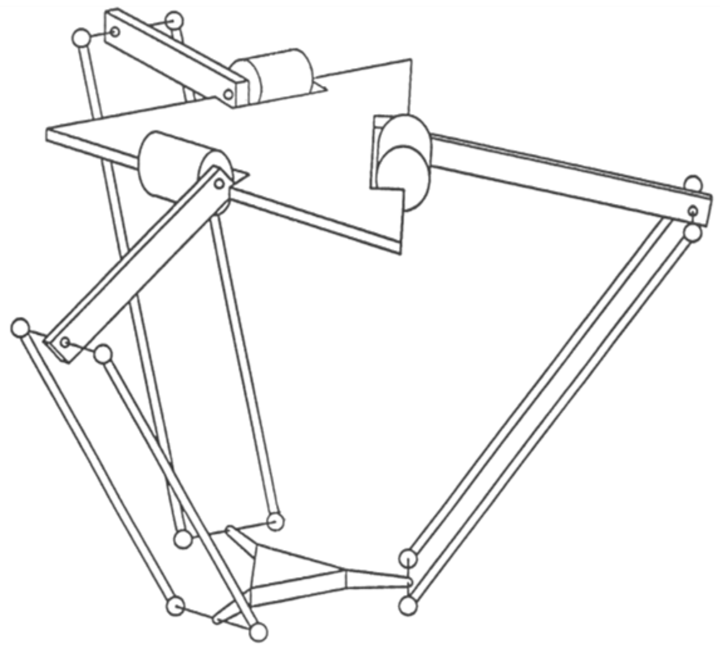
Summary:

- Increases workspace.
- Combines fine and coarse movements.
- Combines fast and slow movements.
- Allows obstacle avoidance in confined spaces.
- Guarantees safety in the event of an actuator failure (requires a particular implementation of the redundancy of the concerned actuator)



Mobility





- **n = 11** (1+1+3.3) {1 basis + 1 mobile plate + 3 arms + 6 bars}.
- **k =15** {(1 pivot + 4 spherical joints) X 3 identical links}.

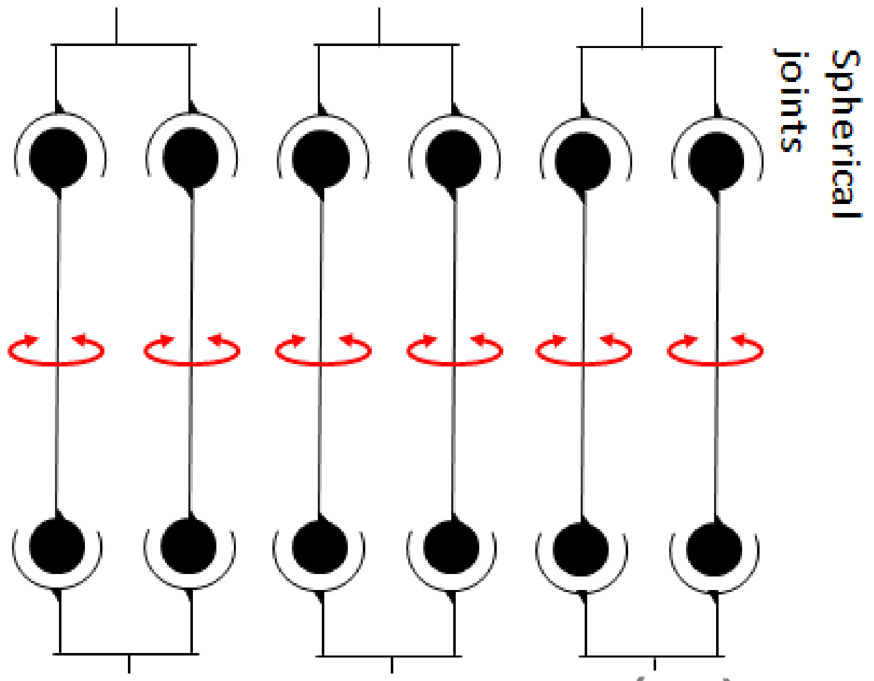
$$\boldsymbol{MO} = 6.(n-k-1)+\sum Mo$$

$$= 6.(11-15-1)+\{1+4.3\}.3$$

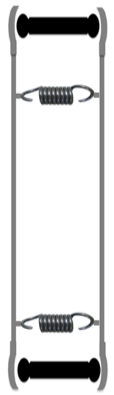
$$= -30+39 = 9$$

$$\boldsymbol{MO} = \sum Mo \ -6.bo = 39-6*5 = 9$$

- The Delta robot as designed with the parallel bars and spherical joints has **<u>6 supplementary mobilities</u>**.
- These mobilities concern **internal mobilities** not affecting **the pure translation of the mobile plate**.
- They are actually related to the **rotation of each bar around its principal axis**.

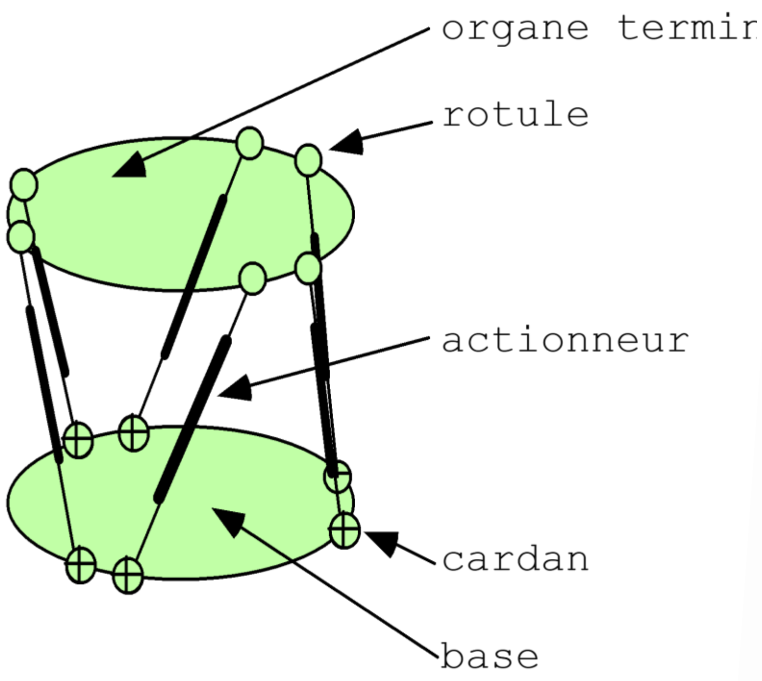


- Springs limit the internal mobility at the cost of friction
- The internal mobility does not affect the final precision of the end effector at the condition of an ideal spherical contact of the spherical joints

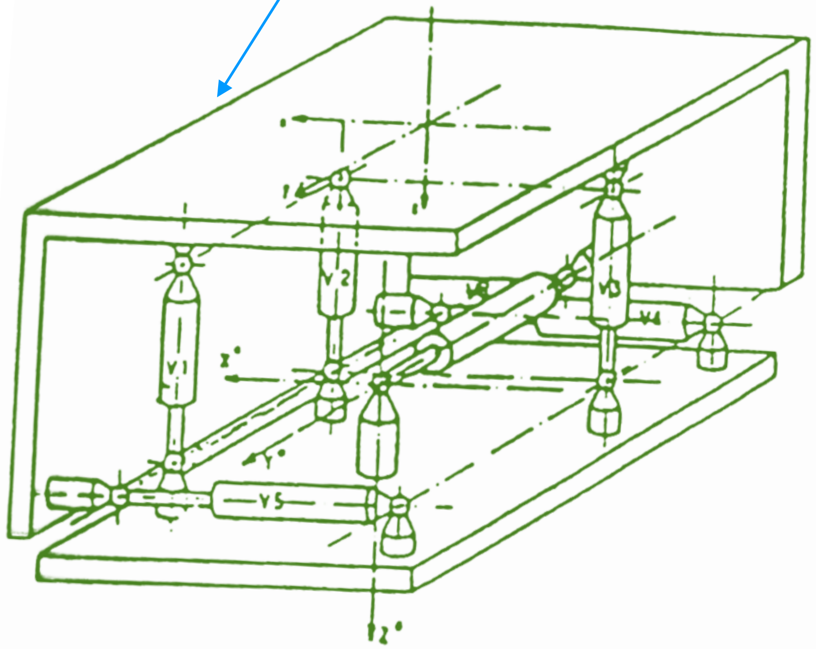


GOUGH-Stewart vs Artigue versus HUNT

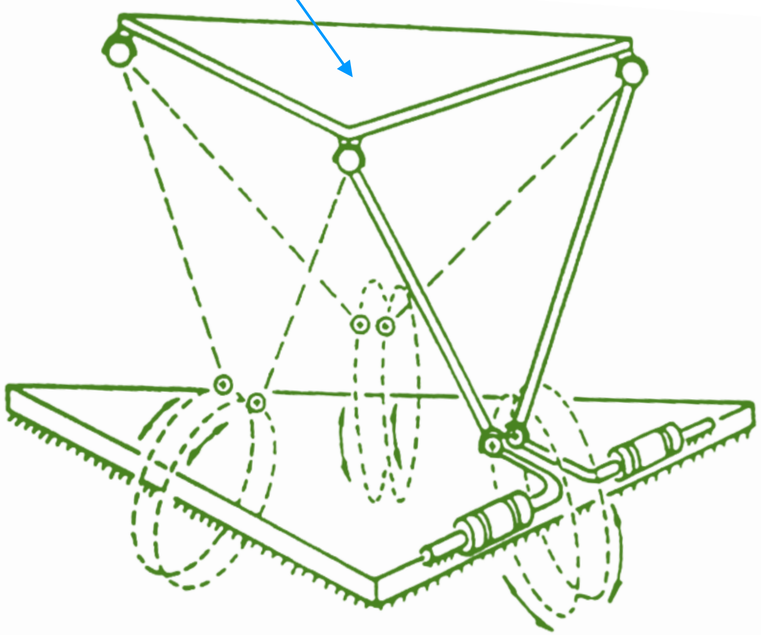
GOUGH-STEWART (1962)



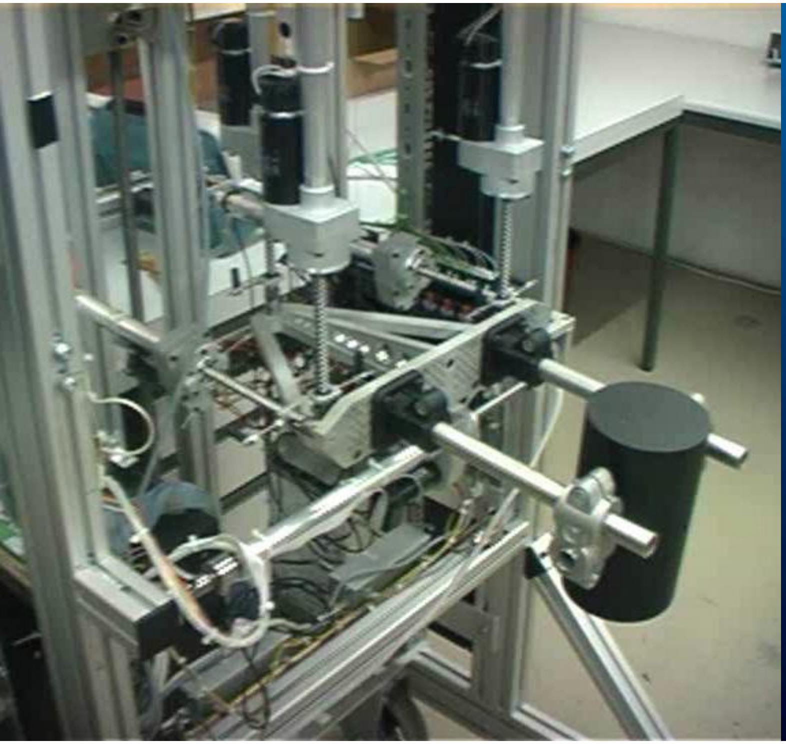
ARTIGUE (1984) and HUNT (1983)



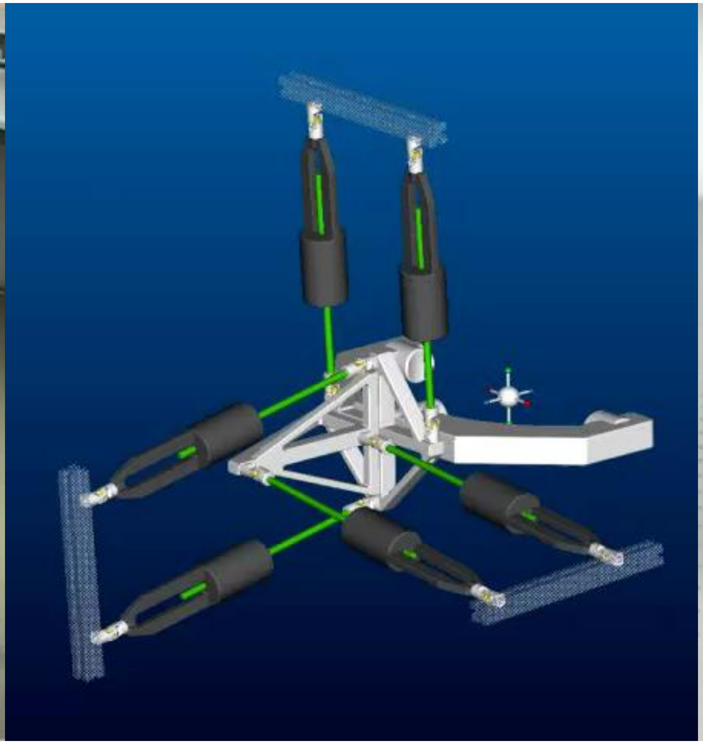
- Decoupled for small motions



- Actuators fixed to base



Artigue 3x2x1



Artigue 2x2x2

Gough-Stewart
Hexapod [ref. Symetrie]

Hunt
Rotational Stewart

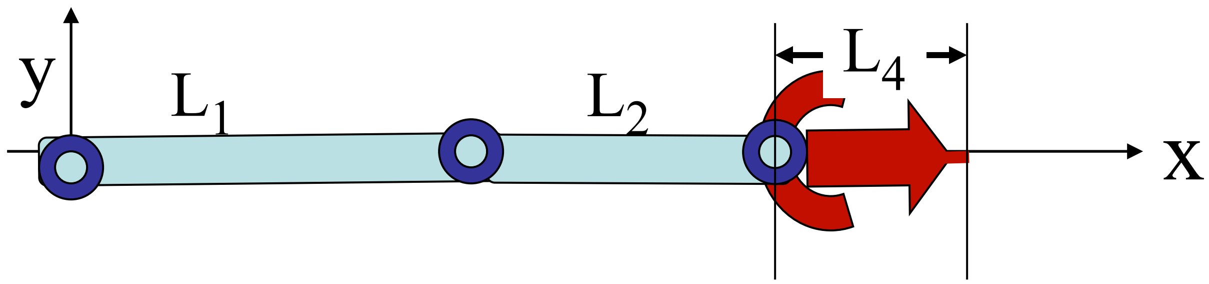
- «**Artigue**» has more **decoupled movements** than «**Gough-Stewart**» and «Hunt»
- «**Artigue 2x2x2**» is **even more decoupled** than «**Artigue 3x2x1**»
- All the **linear variants are stiffer** than the Rotational «Hunt»
- «**Hunt**» is **more dynamic**, has a **bigger workspace** than «**Gough-Stewart**»

2- Kinematics

- Rotation and Homogenous Matrices
- Rotation Matrices and Quaternions to describe rotations
- Different transformations – Matrices of rotations **vs** Axis and Angle of rotation **vs** Quaternions
- Geometric Modelling

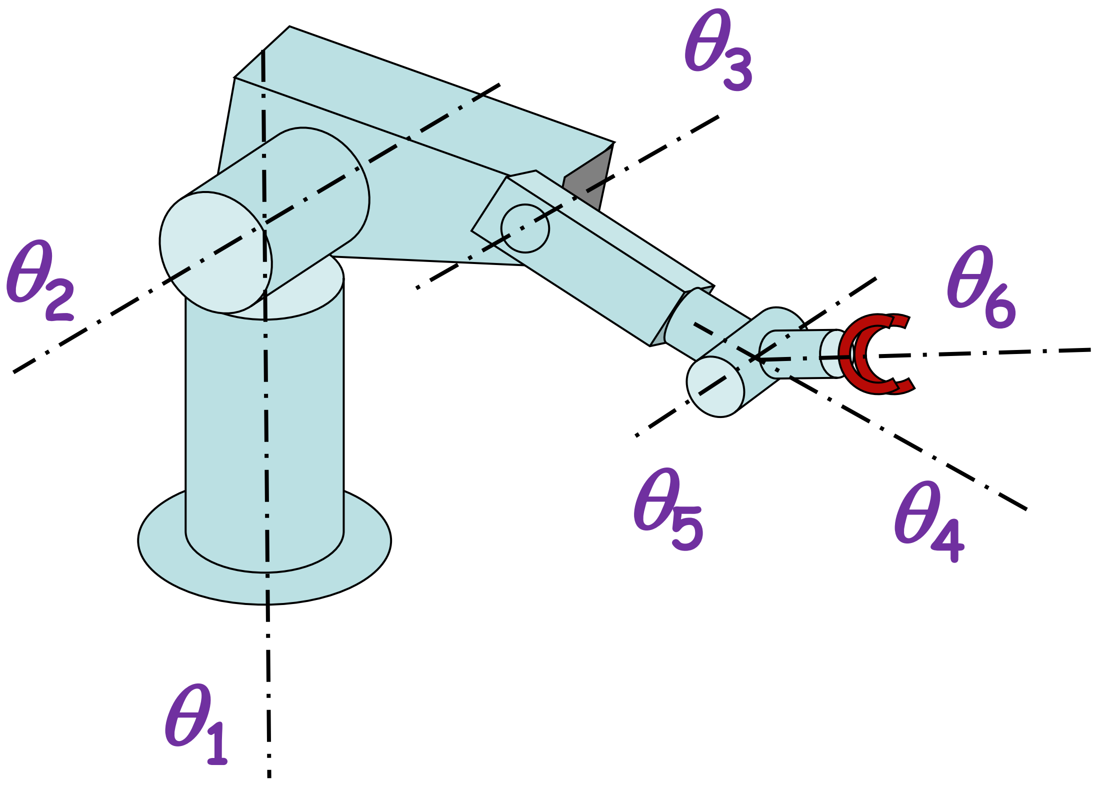
Methodology to establish the DGM in 4 steps

1. Define the **joint variables**
2. Define their **positions of reference**
3. Define the **geometric parameters of the robot**
4. **Chain** the serially-linked successive movements (multiplication of the consecutive homogenous matrices) **starting from the end to the base**



DGM of a 6 DOF robot

1. Joint variables (robot variables)

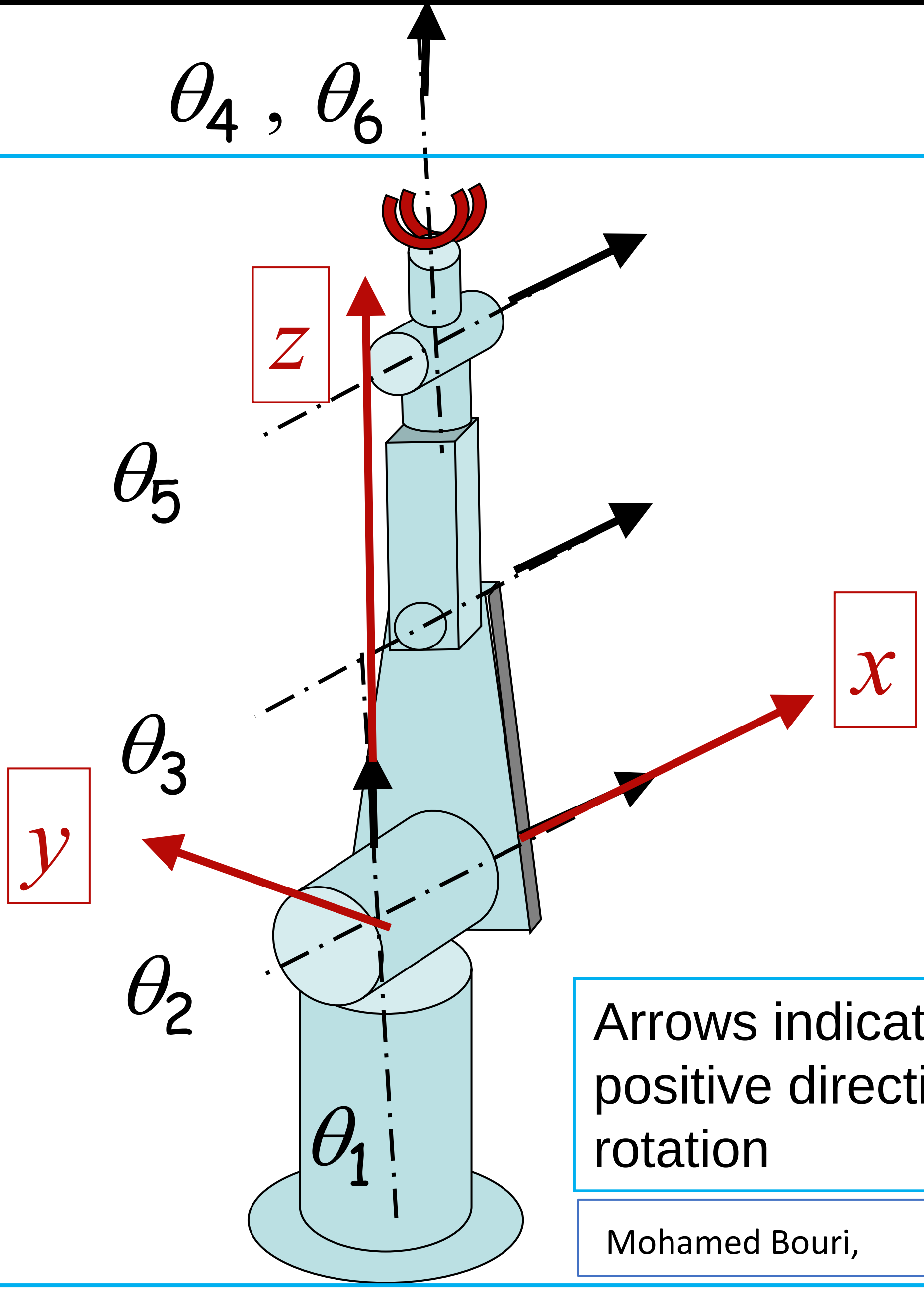


Convention:
Start indexing from the base (frame) to the wrist

DGM of a 6 DOF robot

2. Positions of reference

$$\theta_i = 0$$



The referential {x, y, z} is fixed on the frame (operational coordinates)

Arrows indicate positive direction of rotation

3. Robot parameters

The axes of θ_1 and θ_2 intersect => $\mathbf{L_1 = 0}$

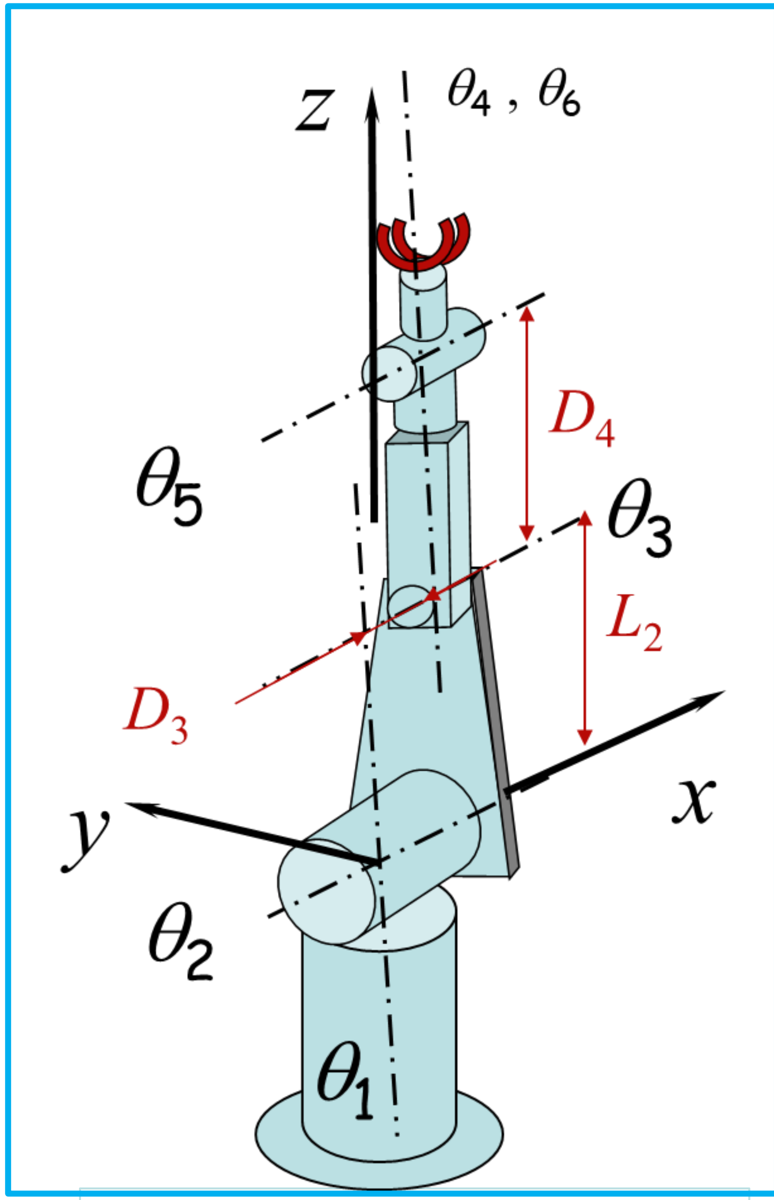
The axes of θ_2 and θ_3 are parallel, dist. $\mathbf{L_2}$

The axes of θ_3 and θ_4 intersect => $\mathbf{L_3 = 0}$

The axes of θ_1 and θ_4 are offset on the axis θ_3 by a distance $\mathbf{D_3}$

The axes of θ_4 and θ_5 intersect => $\mathbf{L_4 = 0}$

The axes of θ_3 and θ_5 are offset on the axis θ_4 by a distance $\mathbf{D_4}$



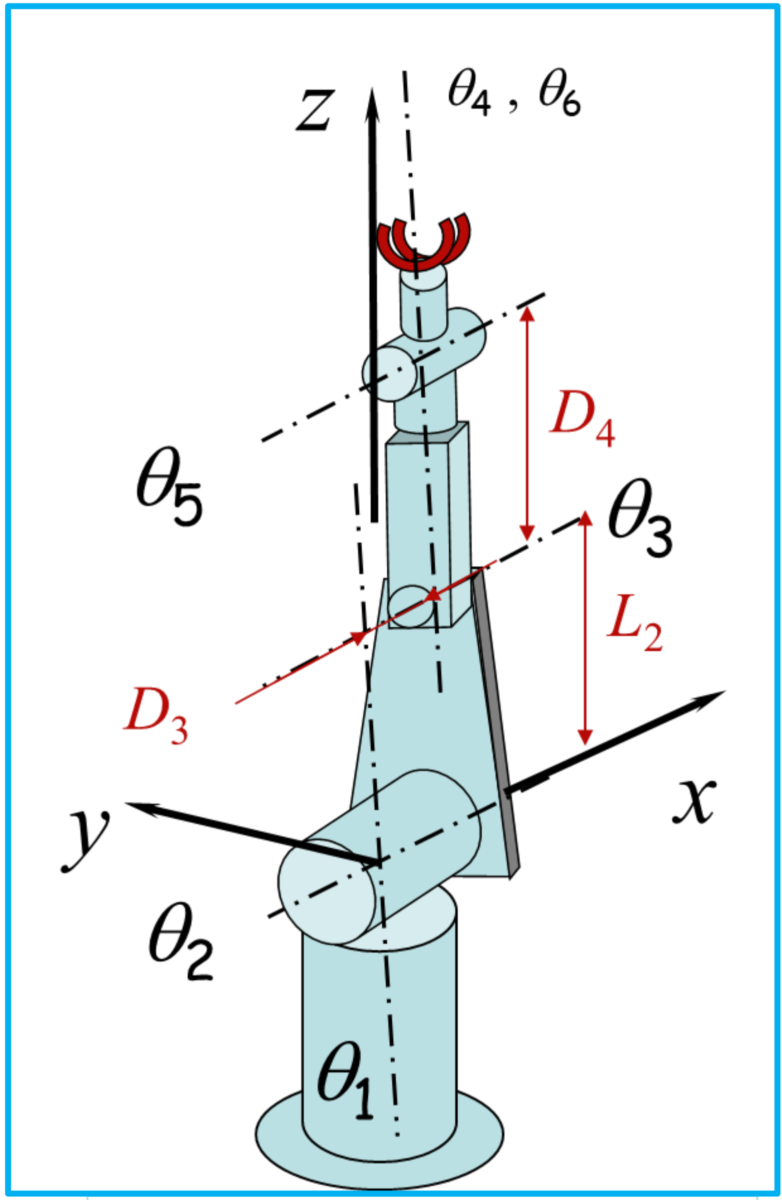
DGM of a 6 DOF robot

4. Sequence of movements

4.1. Rot. of θ_6 around the axis z, distant at $\mathbf{p} = [D_3, 0, 0]'$

$$\mathbf{p} - \mathbf{R}\mathbf{p} = \begin{bmatrix} D_3 \\ 0 \\ 0 \end{bmatrix} - \underbrace{\begin{bmatrix} c_6 & -s_6 & 0 \\ s_6 & c_6 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{R_z} \begin{bmatrix} D_3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} D_3 v_6 \\ -D_3 s_6 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{R} & & \mathbf{p} - \mathbf{R}\mathbf{p} \\ 0 & 0 \quad 0 & 1 \end{bmatrix} = \begin{bmatrix} c_6 & -s_6 & 0 & D_3 v_6 \\ s_6 & c_6 & 0 & -D_3 s_6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \mathbf{K}_6$$



DGM of a 6 DOF robot

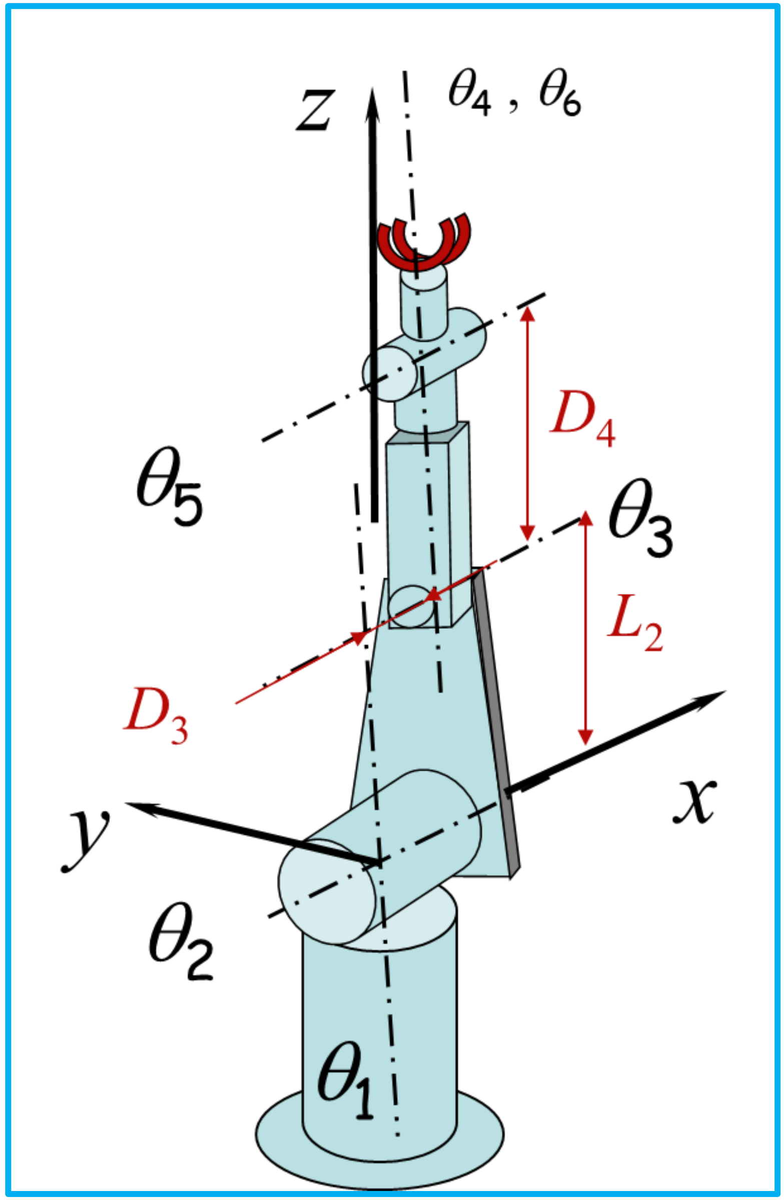
4. Sequence of movements

4.2. Rot. of θ_5 around the axis x, distant at $\mathbf{p = [0, 0, L_2+D_4]'}$

$$\mathbf{p} - \mathbf{Rp} = \begin{bmatrix} 0 \\ 0 \\ L_{24} \end{bmatrix} - \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_5 & -s_5 \\ 0 & s_5 & c_5 \end{bmatrix}}_{R_x} \begin{bmatrix} 0 \\ 0 \\ L_{24} \end{bmatrix} = \begin{bmatrix} 0 \\ L_{24}s_5 \\ L_{24}v_5 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{R} & \mathbf{p} - \mathbf{Rp} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_5 & -s_5 & L_{24}s_5 \\ 0 & s_5 & c_5 & L_{24}v_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \mathbf{K}_5$$

and so on for K4, K3, K2, K1



DGM of a 6 DOF robot

4. Sequence of movements

4.2. Rot. of θ_5 around the axis $\underline{x}$, distant at $\mathbf{p = [0, 0, L_2 + D_4]'}$

$$\mathbf{p} - \mathbf{R}\mathbf{p} = \begin{bmatrix} 0 \\ 0 \\ L_{24} \end{bmatrix} - \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_5 & -s_5 \\ 0 & s_5 & c_5 \end{bmatrix}}_{R_x} \begin{bmatrix} 0 \\ 0 \\ L_{24} \end{bmatrix} = \begin{bmatrix} 0 \\ L_{24} s_5 \\ L_{24} v_5 \end{bmatrix}$$

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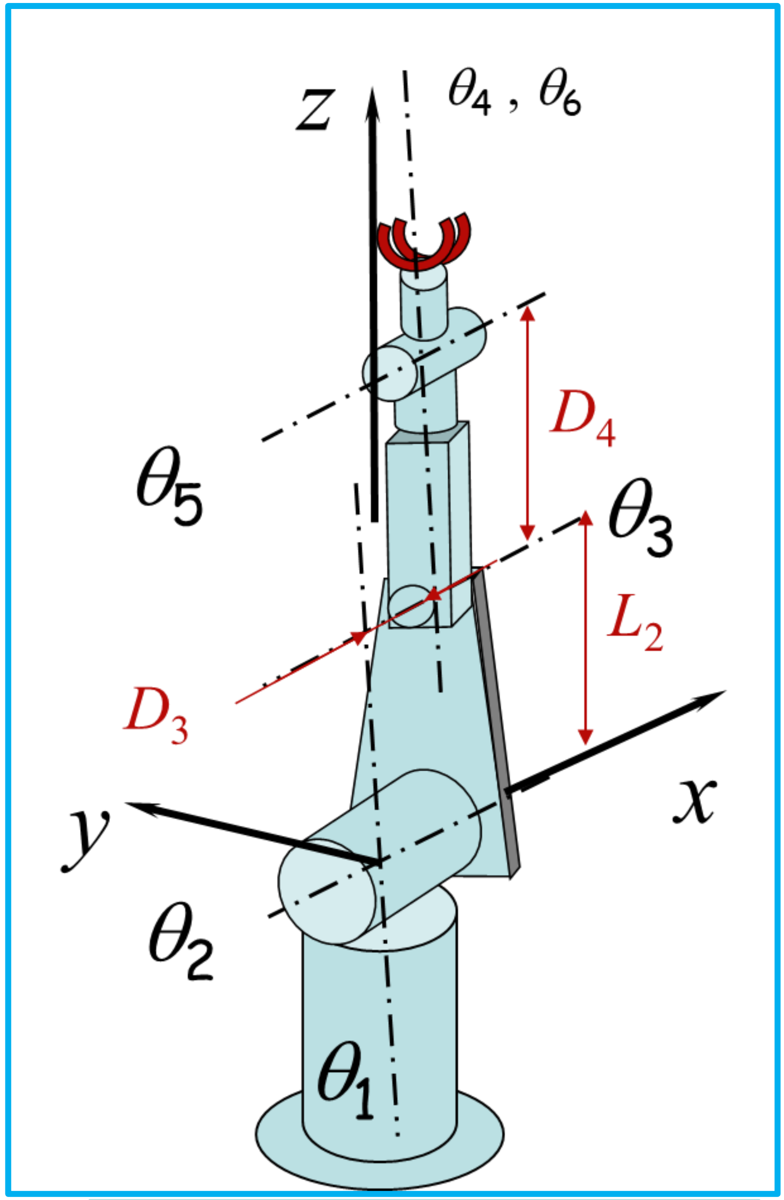
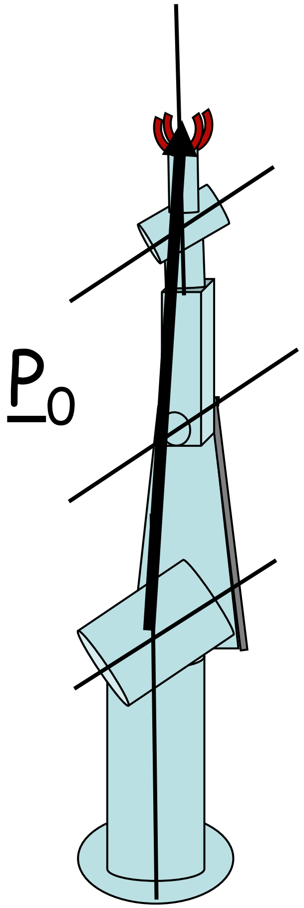
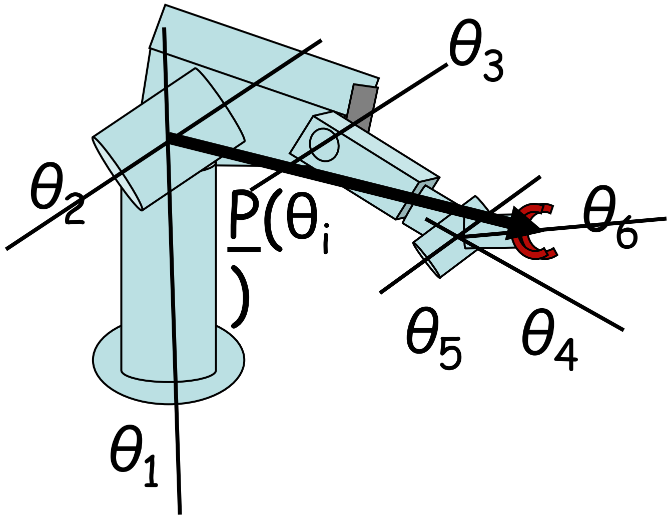


DGM of a 6 DOF robot





Direct geometric model of the robot:

$$P(\theta i) = (K1\ K2\ K3\ K4\ K5\ K6\ )\ P_0$$

Develop all the 6 homogeneous matrices Ki

3 - Differential Kinematics (Jacobian)

Direct Geometric Model

$$X(q) = \begin{bmatrix} X_1(q) \\ X_2(q) \\ \vdots \\ X_m(q) \end{bmatrix} = \begin{bmatrix} f_1(q_1, & q_2, & \cdots & q_n) \\ f_2(q_1, & q_2, & \cdots & q_n) \\ & & \vdots & \\ f_m(q_1, & q_2, & \cdots & q_n) \end{bmatrix}$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} & \cdot & \frac{\partial f_1}{\partial q_n} \\ \frac{\partial f_2}{\partial q_1} & \frac{\partial f_2}{\partial q_2} & \cdots & \frac{\partial f_2}{\partial q_n} \\ & \vdots & & \\ \frac{\partial f_m}{\partial q_1} & \frac{\partial f_m}{\partial q_2} & \cdots & \frac{\partial f_m}{\partial q_n} \end{bmatrix}$$

$$\mathbf{dX} = J \cdot \mathbf{dq}$$

$$\dot{X} = J \ \dot{q}$$

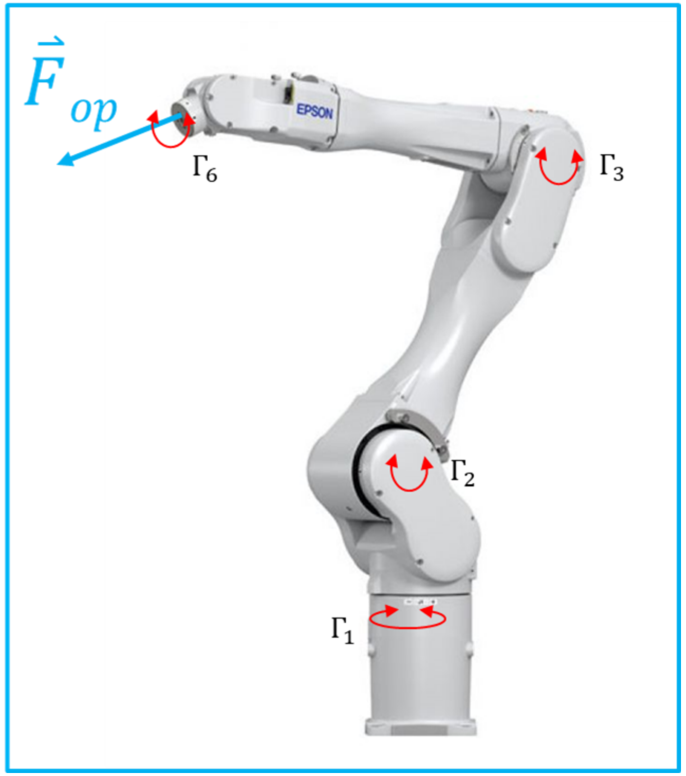
Relationship between Joint and Operational Forces

(Input / output power equality, assumes efficiency = 1)

$$= F_{op}^T . J . \dot{q} = \Gamma_{art}^T . \dot{q}$$

$$\Rightarrow F_{op}^T . J = \Gamma_{art}^T$$

$$\Rightarrow J^T . F_{op} = \Gamma_{art}$$



3 - Differential Kinematics (Jacobian)

1. It links the joint speeds to the operational ones, it gives important information on the nominal and maximum speeds of the motors to be selected. It is a **reduction or transmission matrix**.

2. It is also called the **sensitivity matrix** because it is allowing to know the sensitivity at the output level knowing those of the joints (motors + local transmission).

3. It allows the projection of forces between the joint and the tool spaces,

4. It allows to know and control the singularities of the robot. It is also called a **stability matrix**.

5. It allows to **numerically invert the** direct **geometric model** (ie. By using the Newton-Raphson method) to obtain the inverse geometric model (or vice versa).

6. It can be used in several **control approaches**.

7. Useful for dynamic modeling.

4 – Dynamic modeling

$$\Gamma = B(q).\ddot{q} + G(q) + C(q,\dot{q}) + F(q,\dot{q}) + K(q)$$

1. Describes the behavior of the required joint torques, function of the joint poistions, velocities, and accelerations.

2. Is obtained using the **Lagrange** Approach (systematic) or **Newton-Euler** approach (requires to write the set of equations, one for each robot sub-body)

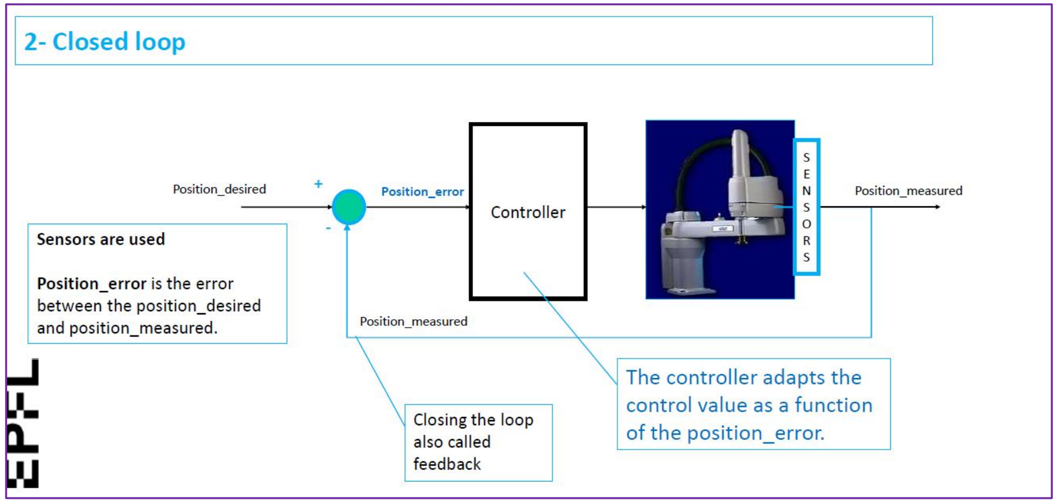
3. Is used to simulate the dynamic behavior and be more immersive with real dynamic conditions.

4. Helps to size the robot bodies and actuators

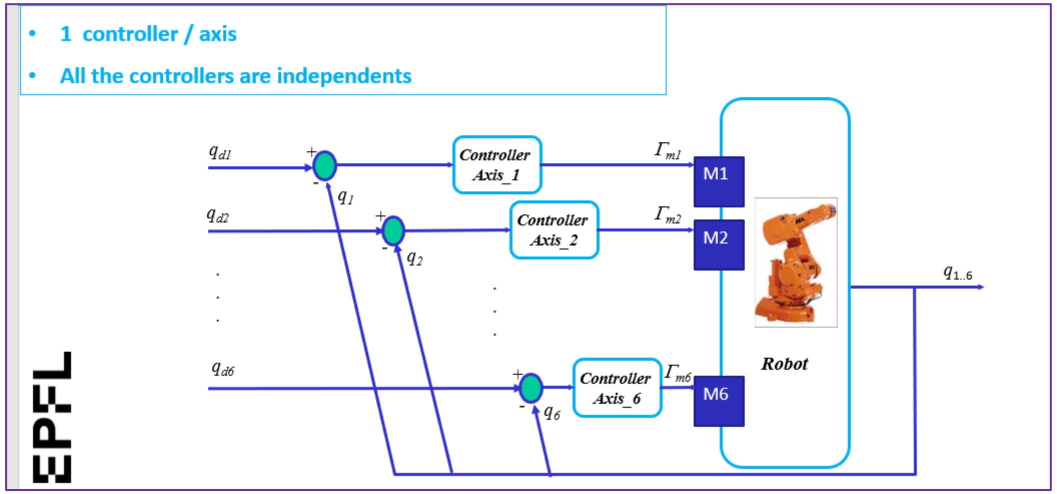
5. Develop and implement conrol strategies.

5 – Control

1. The role of a controller is to make the output (position) reach a defined target.



2. Decentralized controllers (convential) consider each actuated axis of the robot as independent (PID or cascaded P (position) , PI (velocity))



Dynamic couplings are considered as disturbances.

Please remind me about the role of **P, D and I parameters**

Proportional gain (Kp)	**Derivative gain (Kd = Kp * Td)**	**Integrator gain (Ki = Kp / Ti)**
$\Gamma_p = K_p.e$, proportional to the position error e : It **acts as a spring**	$\Gamma_d = -K_d.\frac{d\theta}{dt}$, conversely proportional to the velocity : It **acts as a viscos friction**	$\Gamma_i = K_i.\int e\ dt$, **more and more effect** up t cancelling the error, estimation function.
Increases the stiffness of the control	Improves the damping in closed loop	Cancels the steady state error (sse) (completely eliminates the sse)
Increases the eigenfrequency in closed loop, $\omega_n \propto \sqrt{\frac{K_p}{J}}$,	Stabilizes the controlled system	Estimates the steady state disturbance at the origin of the sse,
Improves the response time (higher closed loop dynamics)		
Higher proportional gain increases the robustness of the control with respect to parameter and external disturbances		
Reduces (not cancels) the steady state error.		

Please remind me about the **limits** of **P, D and I parameters**

Proportional gain (Kp)	**Derivative gain (Kd = Kp * Td)**	**Integrator gain (Ki = Kp / Ti)**
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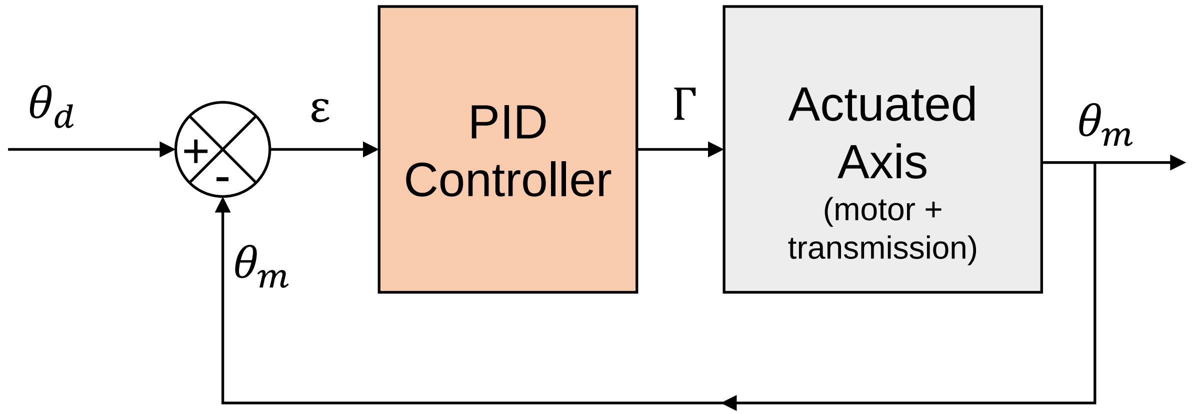
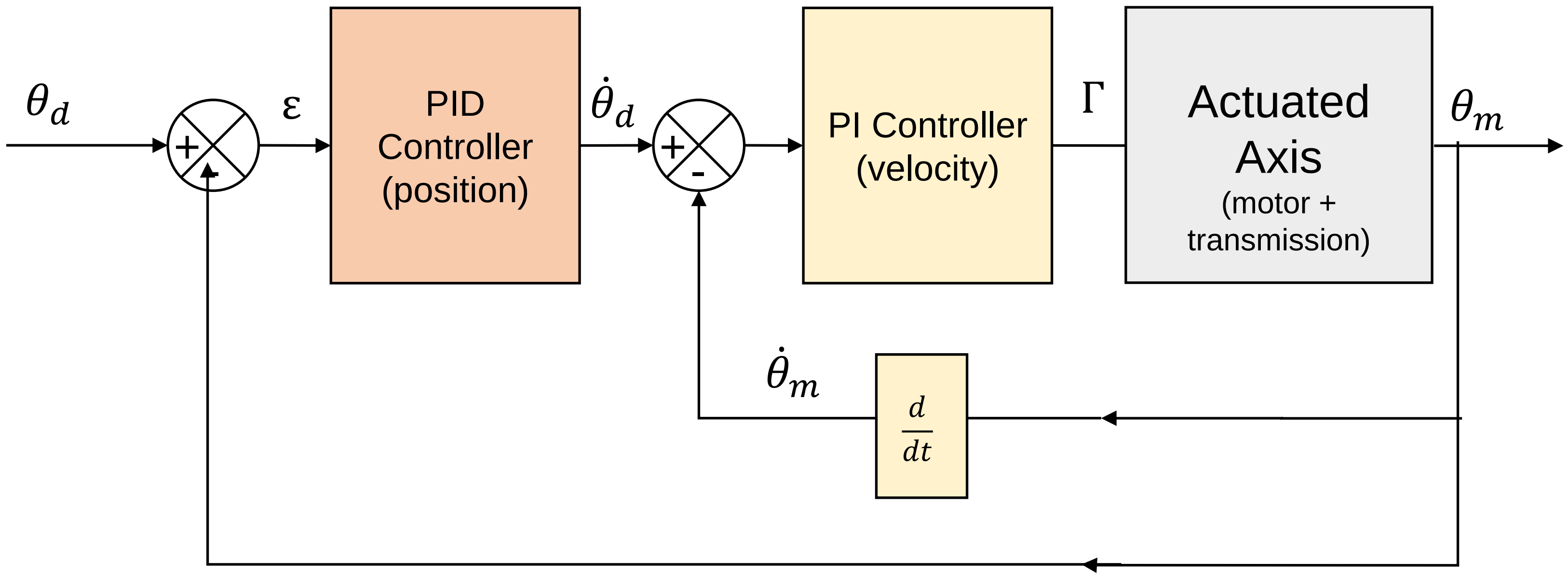
Higher gains saturate the control torque, • the system gets into a non linear area, because of the control saturation. • It may make the position oscillate, notice that we are in an open loop during saturation,	Differentiation is know to amplify the noise, • which increases the noise at the torque control, • Which may excite mechanical resonant frequencies	The integrator affects the convergence time to a complete cancelation of the steady state error.
Higher gains may destabilize the closed loop system (demonstrated with a root locus and Nyquist linear representations)	The viscous effect may induce to slower behaviours.	Increasing the integrator gain, to make the system faster, may create overshoots when loading/unloading the integrator.
	In some cases, we may be motivated by adding filters, or derivate at higher sampling periods, this induces delays that consequently may destabilize the system.	**Advice**, limit the contribution of the integrator to a maximum of 25-30% of the maximum applied torque (use an anti reset windup ARW)

Controllers without models

PID position controller

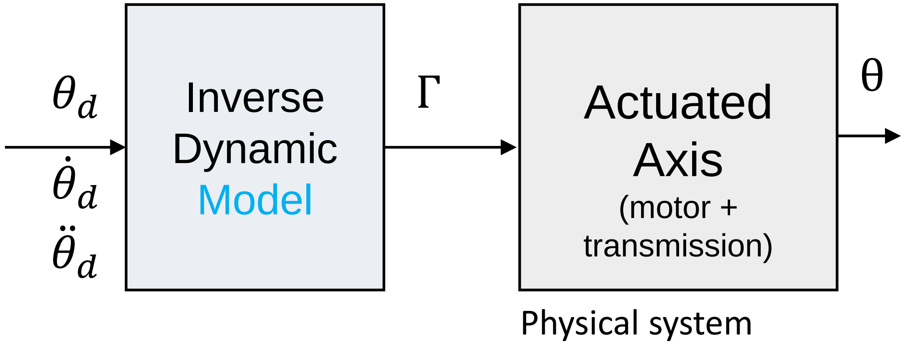


Cascaded controller (position / velocity)

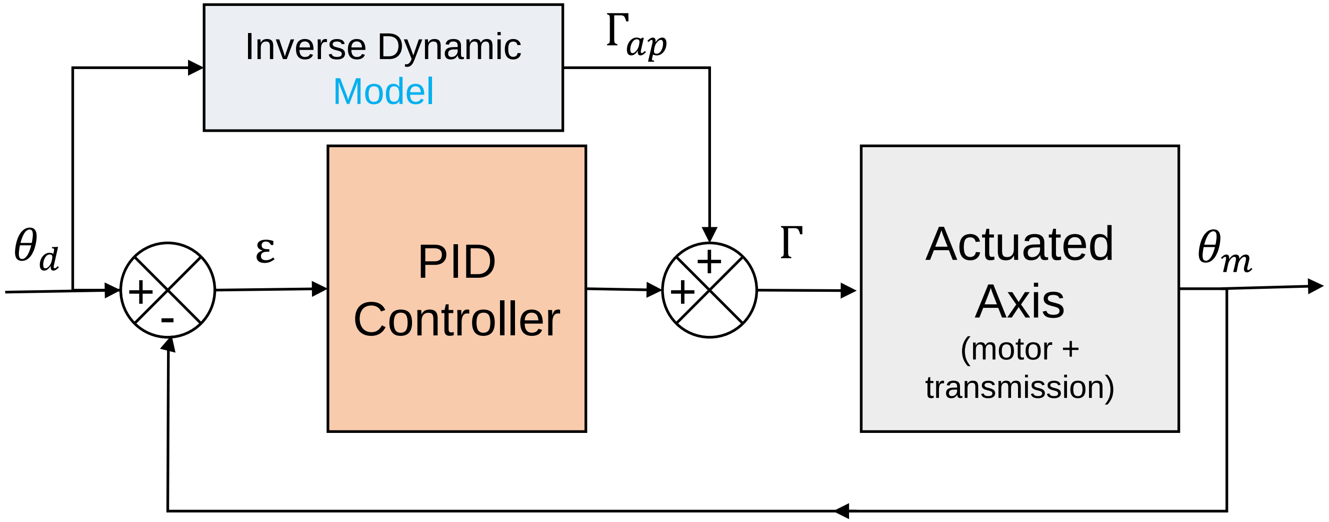


Controllers with models

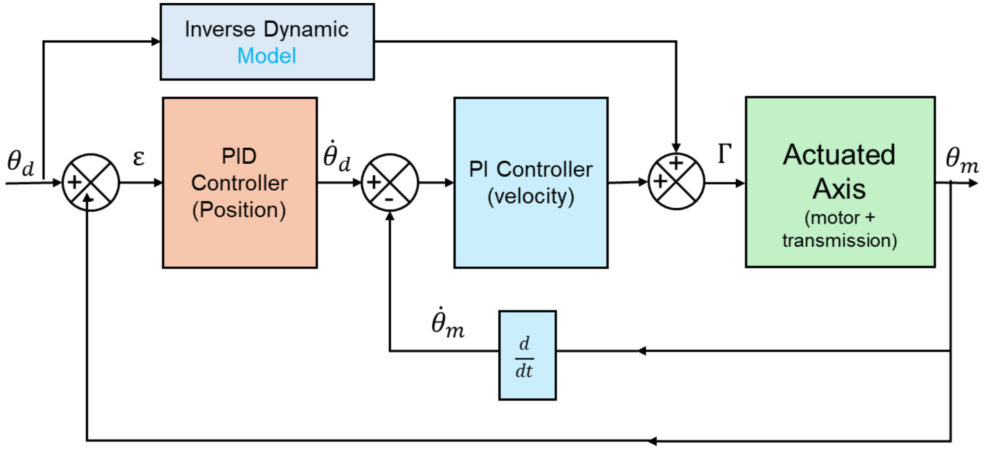
PID position controller



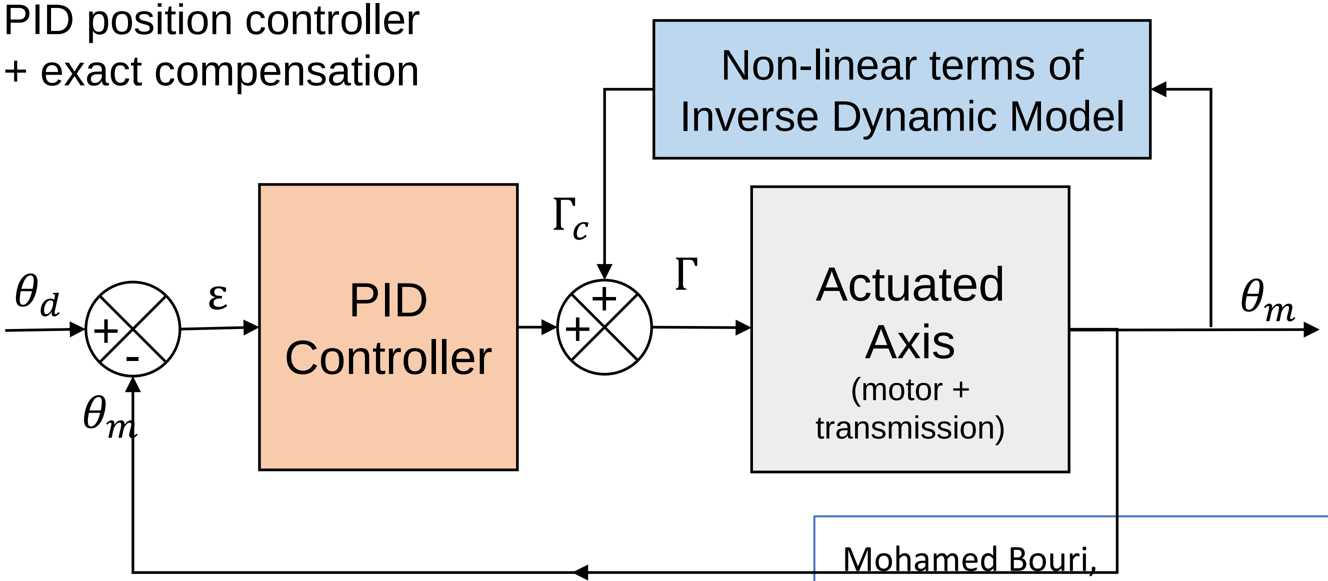
PID position controller + a priori (feed forward)



Cascaded controller (position / velocity) + a priori

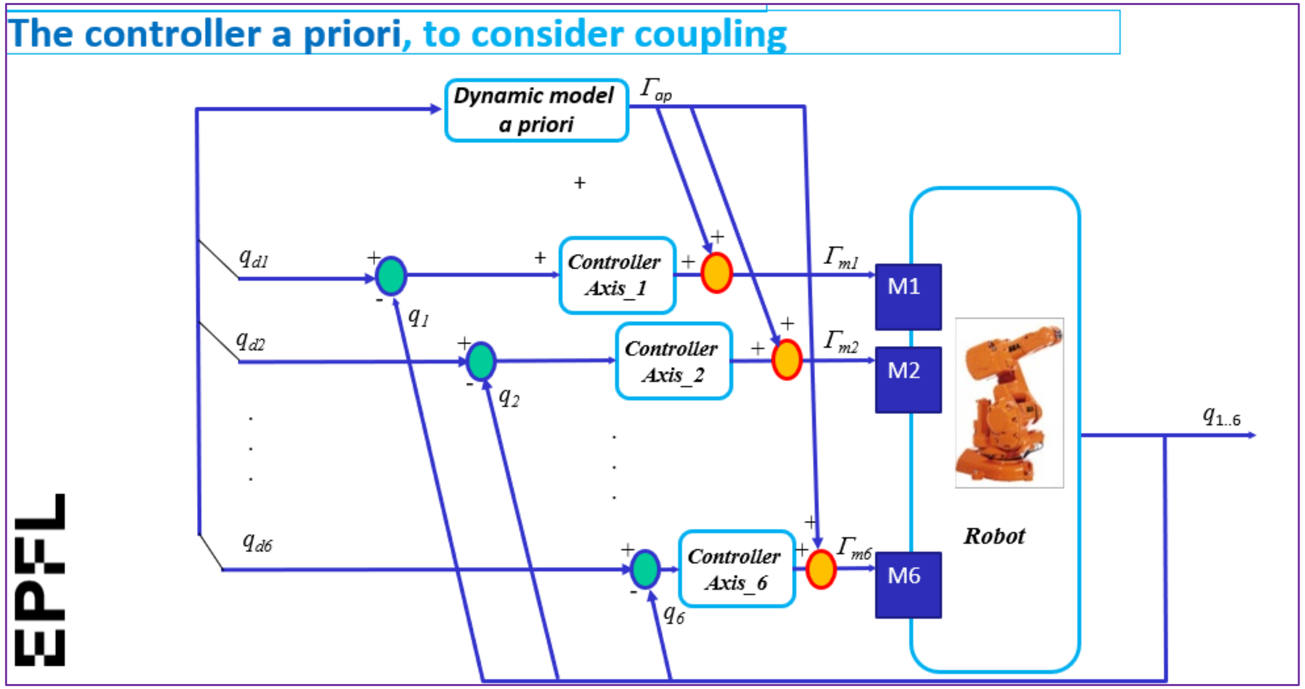


PID position controller
+ exact compensation



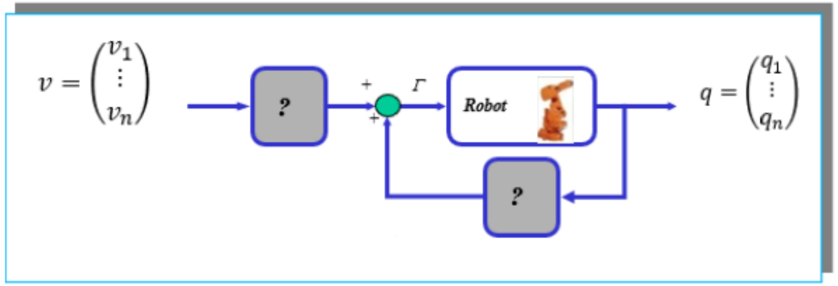
5 – Control

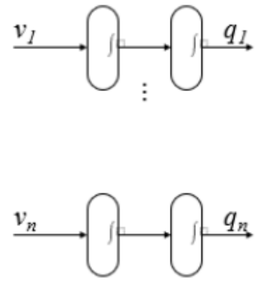
3. The simplest way to consider coupling is to implement a feedforward dynamic compensation (a priroi)



5 – Control

4. The other way to consider coupling is to implement an exact nonlniear compensation, which **1**- linearizes the behaviour and **2**- decouples the dynamic behaviour

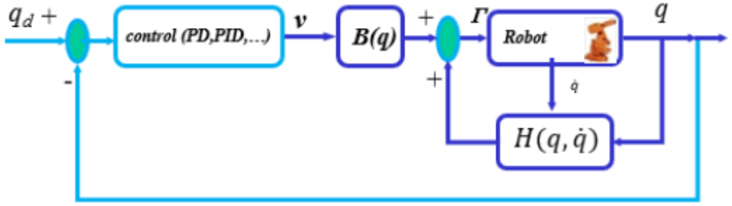




The double integrators must now be stabilized by looping:

- PD,
- PID,
- or advanced controllers (adaptive ,sliding mode, GPC, MPC...)

$$\Gamma = B(q).v + H(q,\dot{q})$$

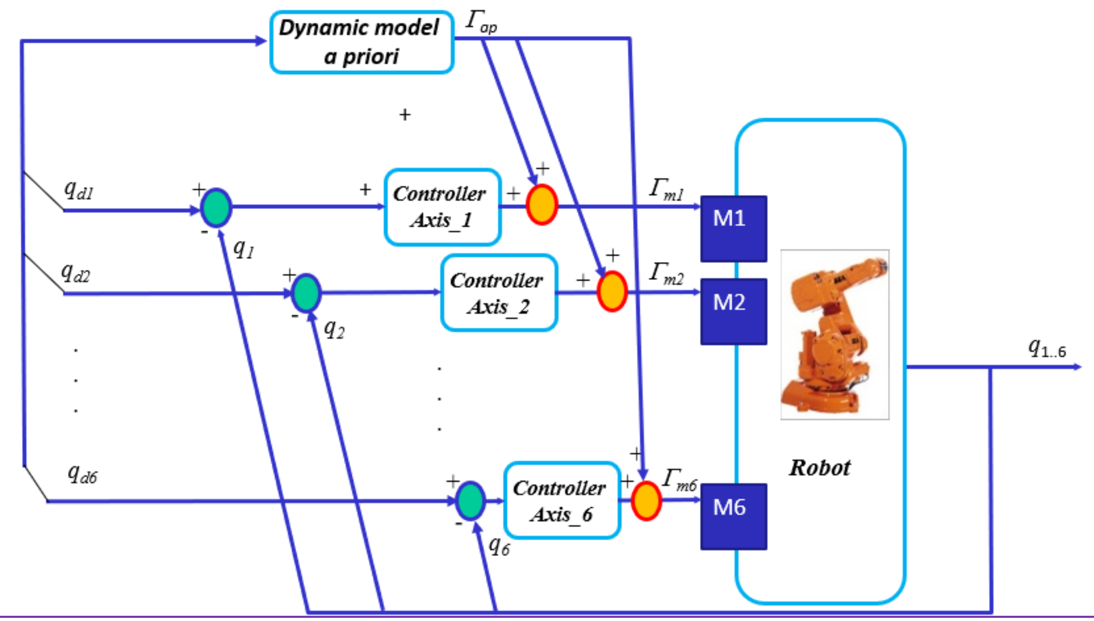


5 – Control

5. The « **torque a priori** » (feed forfward torque) compensates for dynamic and static behaviors.
However, it makes more senses when it compensates for dynamic effects (terms proportional to accelerations and velocities).

For **steady state compensation**, the integrator of the PID, contributes to cancel the static error.

The controller a priori, to consider coupling



EPFL

6 – Actuators

1. Electrical actuatotrs are the most adopzted for Industrial robotics.
   Electrohydraulic are stll favoured for heavy requiequirements, and very high payloads and forces.
   Electropenumatic actuators are the simplest choice for low cost automation and compact grippers.

2. There are 3 main categories of electrical actuators:
   1- Stepper motors (simplest to control (open loop), low cost, high tprques but lower velocities and noisy.
   2- Brushed DC motors (With IRON: cheap, cost effective but not effeicent – IRONLESS : very efficient)
   3- Brushless DC motors are the most used for industrial robotics, do not suffer any more of the complexity and cost of the control electroonics.

3. Most of the actuators are equipped (already integrated with position senors-

4. The choice of electrical DC motors is made based on :

$$Km = \frac{\Gamma}{\sqrt{R.i^2}} \quad [\mathrm{Nm}/\sqrt{W}]$$

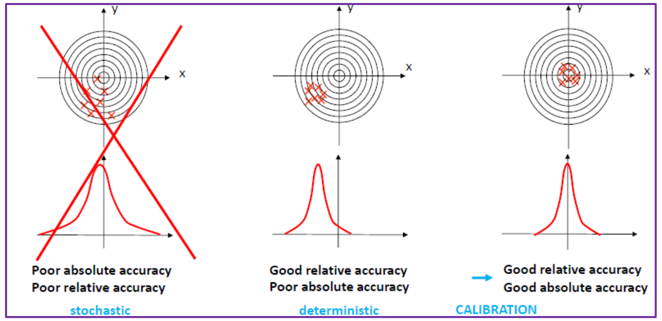
– Nominal and peak torques – Factor of Regulation (K/R^2, the lowest possible) – The motor constant Km (*, the highest) – Dimensions, and footprint – Density of torque (Nm/ kg) -

5. The optimal gear ratio helps to choose the reducer. A bad choice of the gear ration affects the acceleration capabilities-

7 – Sensors

1. Resolution, linearity, and bandwidth are the most important information for sensors.

2. Sensors with good repeatability can be calibrated.



3. Temperature is the biggest enemy of high precision. Count about 10microns of dilation / °C

4. Incremental and absolute encoders are the most used for position sensing in robotics.

5. Velocity is obtained by numerical derivation. Higher is the sampling frequency, worst is the precision of the velocity.

6. A good position resolution, favors a good control stiffness- A good velocity resolution, favors a good stabilization and damping.